AN UNFITTED METHOD WITH ROBIN BOUNDARY CONDITIONS FOR THE ANALYSIS OF HETEROGENEOUS ARTERIAL SECTIONS

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Introduction

Ischaemic heart disease is a frequent cause of death globally, with coronary atherosclerosis being the cause of most myocardial infarctions [1]. Typically, a patient presents either stable or unstable plaque. The fast distinction between these two groups is crucial regarding patient treatment. This work proposes an unfitted immersed boundary (IB)-based methodology [2] with a more physical elastic bed boundary condition (BC) to analyze coronary artery sections undergoing uniform pressure in a quasi-static regime.

Methods

We propose to assume the section to be surrounded by a material along its external boundary. This embedding matrix produces a linear elastic reaction, and it is described with an elastic bed coefficient (**α**) depending on the stiffness. The governing equation for the elastic problem becomes [3,4]:

$$\int_{\Omega} \sigma(u) : \varepsilon(v)\, d\Gamma - \int_{\Gamma_R} \alpha u \cdot v \, d\Gamma = \int_{\Gamma_N} t \cdot v \, d\Gamma \qquad (1)$$

where **σ** is the Cauchy stress tensor, **u** the displacement field, **ε** the strain field, **t** the surface traction, and **v** a virtual infinitesimal displacement. $\mathbf{\Gamma_R}$ represents the elastic-bed BC, equivalent to the well-known Robin BC, while $\mathbf{\Gamma_N}$ the Neumann BC. To improve the computational efficiency, the proposed methodology implements (1) in an IB framework with a generic description of the domain, Ω, based on level-sets. The level sets (Fig. 1), defined on a fixed background mesh, provide an implicit description of arterial sections by dividing the domain into subdomains corresponding to different plaque components (e.g., healthy, fibrous, lipid, and calcified core). Without a loss of generality, linear elastic behavior is assumed for all components.

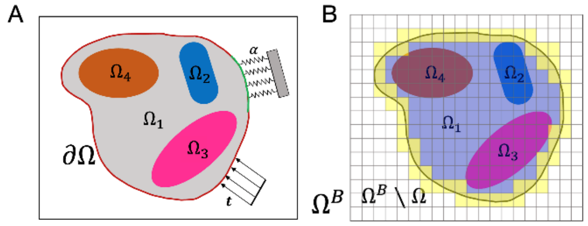

Figure 1: (A) Problem 1 description, with Γ_N in red and Γ_R in green, and (B) background mesh with level-sets.

Results

To show the accuracy of the methodology, problem (1) is solved using the proposed IB Robin-based (IBR) model, on a realistic coronary section (Fig. 2A) subjected to an internal pressure p = 1e-2 MPa, and α = -1.77e-4 MPa/mm. Figure 2B shows the displacement field obtained with the proposed IB method, while Figure 2C corresponds to the solution obtained with the classical Finite Element (FE) method. Differences in the average error of the displacement magnitude are found to be less than 0.5% in the section, with a maximum difference of less than 5% (Fig. 2D).

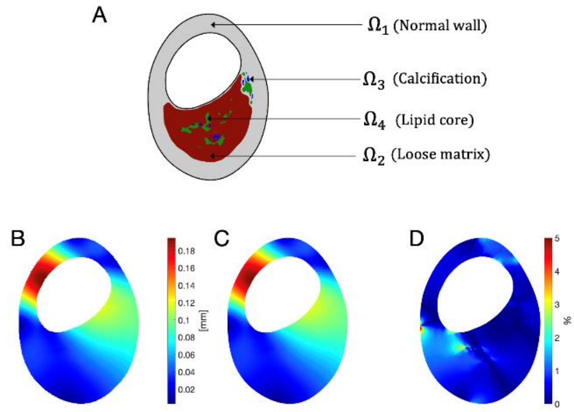

Figure 2: IBR (B) and FE (C) displacement, with real coronary section (A) and the relative local error (D).

Discussion

The article presents a novel formulation that combines hierarchical level sets (from a 2D arterial segmentation) with an IBR-based formulation to obtain stress and strain fields in arterial sections under physiological conditions of blood pressure. The level-sets allow us to describe the arterial geometries, including plaque component distributions, and use a single background mesh to simulate patient-specific arterial segmentations, avoiding to develop a different conformal FE mesh per geometry. Using the Robin BC (instead of classical Dirichlet) allows to remove the rigid body motion without altering the natural deformation of the arterial section, and to account for the effect of the surrounding tissue on the artery. The results on realistic coronary arterial sections demonstrate that the proposed unfitted IB-based approach provides results equivalent to the standard FE, allowing seamless integration of structural analysis in a medical image processing pipeline.

References

1. Thygesen et al, J European Heart, 28:2525-2538, 2007.
2. Peskin, Acta Numerica, 11:479-517, 2002.
3. Teng et al, Acta Biomaterialia, 10:5055-5063, 2014.
4. Ebenstein et al, J Biom. Mat. Res., 91A:1028-1037, 2009.