CLASS Tutorial

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CLASS overview

- CLASS (The Cosmic Linear Anisotropy Solving System) is an Einstien-Boltzmann code for simulating the evolution of linear perturbations in the universe.
- CLASS can be used to calculate the cosmological observables within the framework of the standard LCDM cosmological model and beyond.
- It can be used to perform detailed calculations of the Cosmic Microwave Background (CMB) power spectrum, as well as other large-scale structure observables.
- CLASS is very structured, user-friendly, and flexible to modify.
- CLASS was written by Julien Lesgourgues \& Thomas Tram, first released in 2011.
- **Ref:** Lesgourgues, J. (2011) **[1104.2932, 1104.2933]**

For more information about CLASS can be found on the website:

http://class-code.net

What does CLASS do?

- Solves for the nonlinear, coupled Einstein-Boltzmann equations for many types of cosmic component in the Universe to first order in perturbations.
- Computes the CMB observables such as temperature and polarisation power spectrum such as $C_\ell^{TT}, C_\ell^{TE}, C_\ell^{EE}, C_\ell^{BB}$
- Computes Large Scale Structures (LSS) observables such as the total matter power spectrum *P*(*k*) and individual density and velocity transfer functions.

What can you get from CLASS?

Background evolution

The solutions of matter and radiation are given by

$w = 0$ for dust

$w = \frac{1}{3}$ for radiation

$w = -1$ for the cosmological constant

$w = w(a)$ for dynamical dark energy e.g.

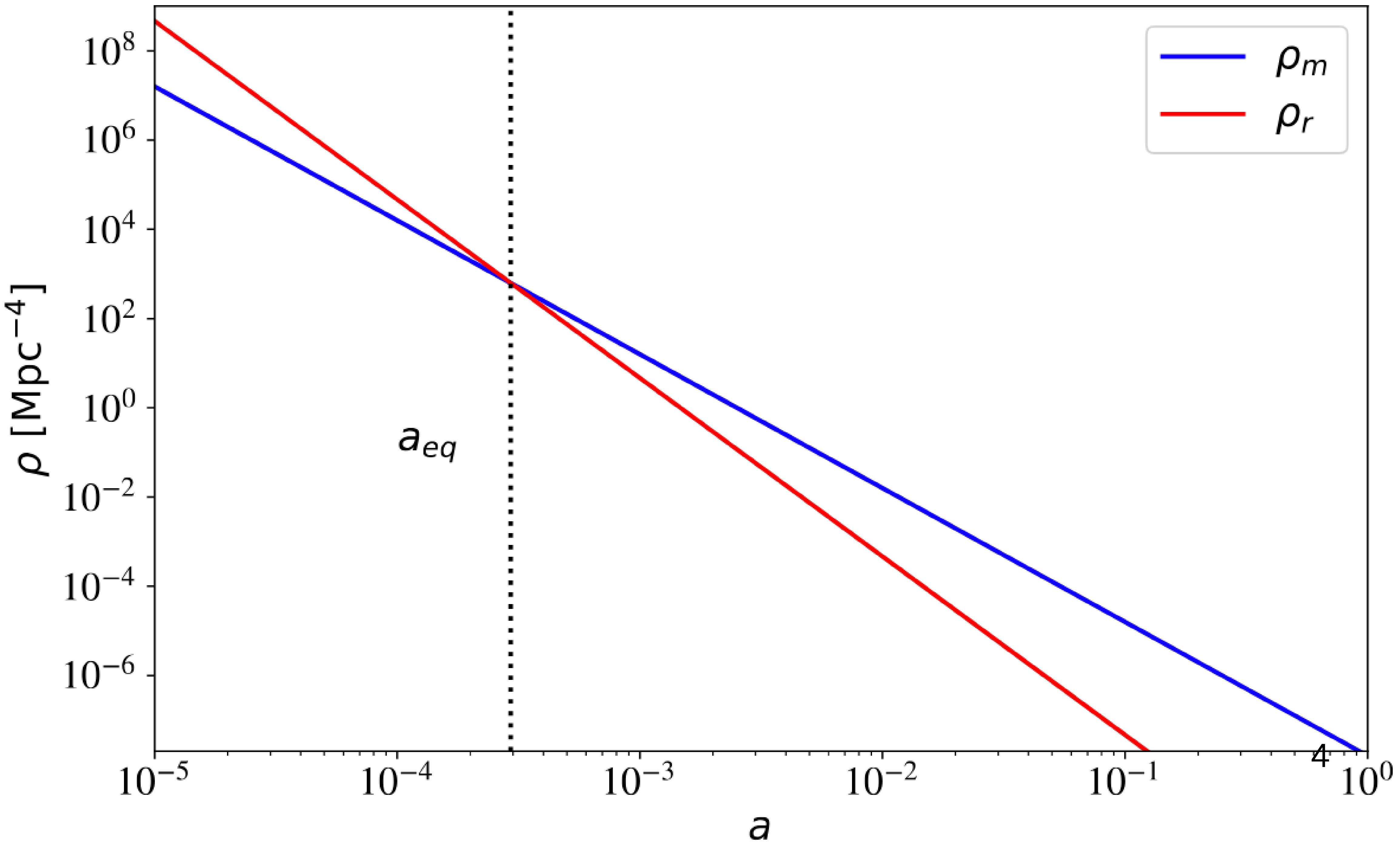
$w(a) = w_0 + w_a(1 - a)$ (CPL model)

Continuity equation

$$\dot{\rho} = -3H\rho(1 + w)$$

$$\rho_m = \rho_{m,0} a^{-3}$$

$$\rho_r = \rho_{r,0} a^{-4}$$



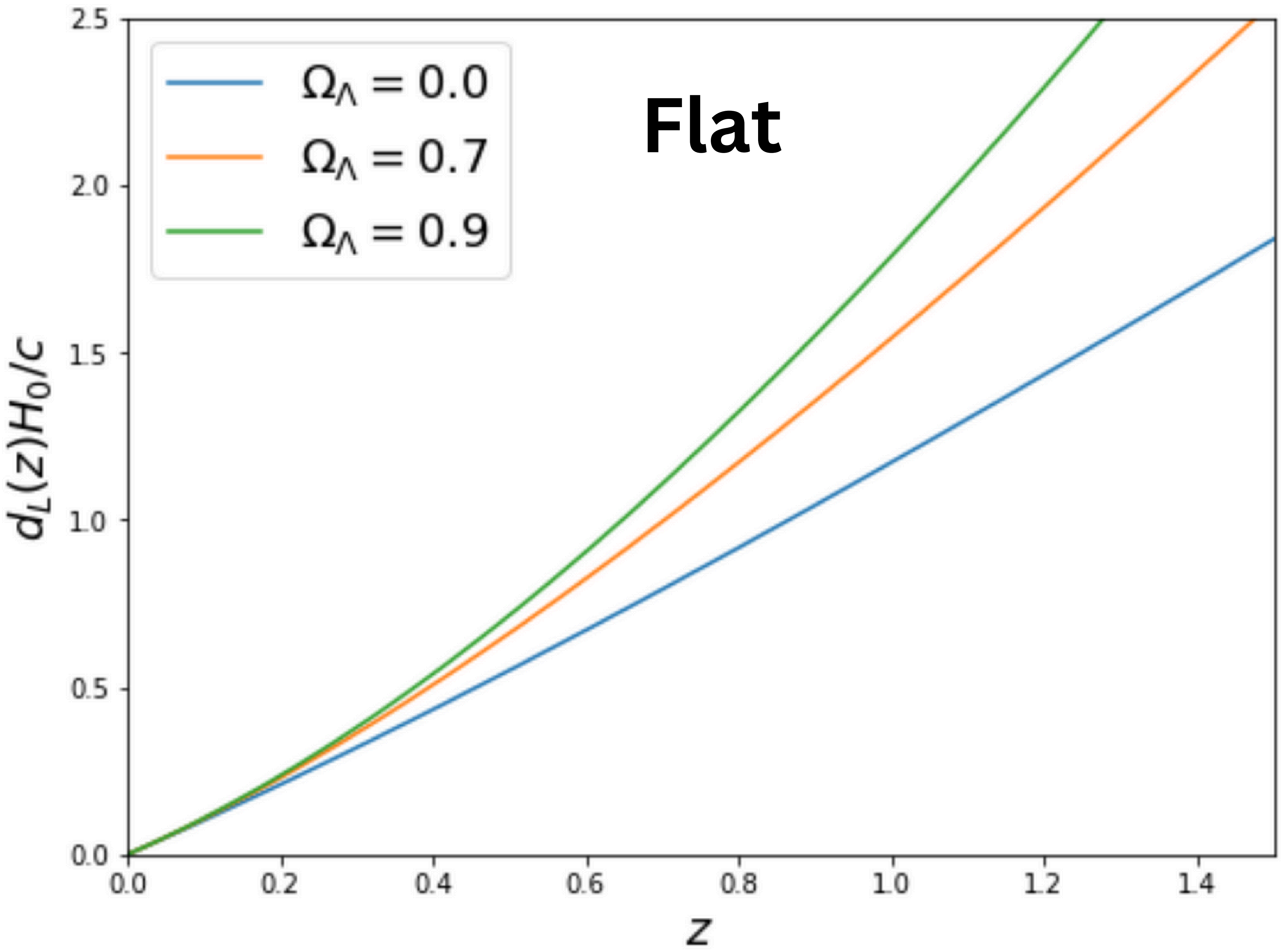
cosmic distances

Luminosity distance

$$d_L = \frac{(1+z)c}{H_0\sqrt{\Omega_{k,0}}}\sinh\left[\sqrt{\Omega_{k,0}}\int_0^z \frac{dz'}{E(z')}\right]$$

in case of flat space

$$d_L = \frac{(1+z)c}{H_0}\int_0^z \frac{dz'}{E(z')}$$



where

$$E(z) = \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]^{1/2}$$ is from the Friedmann equation

$$E(z) \equiv \frac{H(z)}{H_0}, \quad \Omega_\alpha \equiv \frac{8\pi G}{H_0^2 c^2}\rho_\alpha, \quad \Omega_{k,0} \equiv -\frac{k\,c^2}{H_0^2 a^2}$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k\,c^2}{a^2}$$

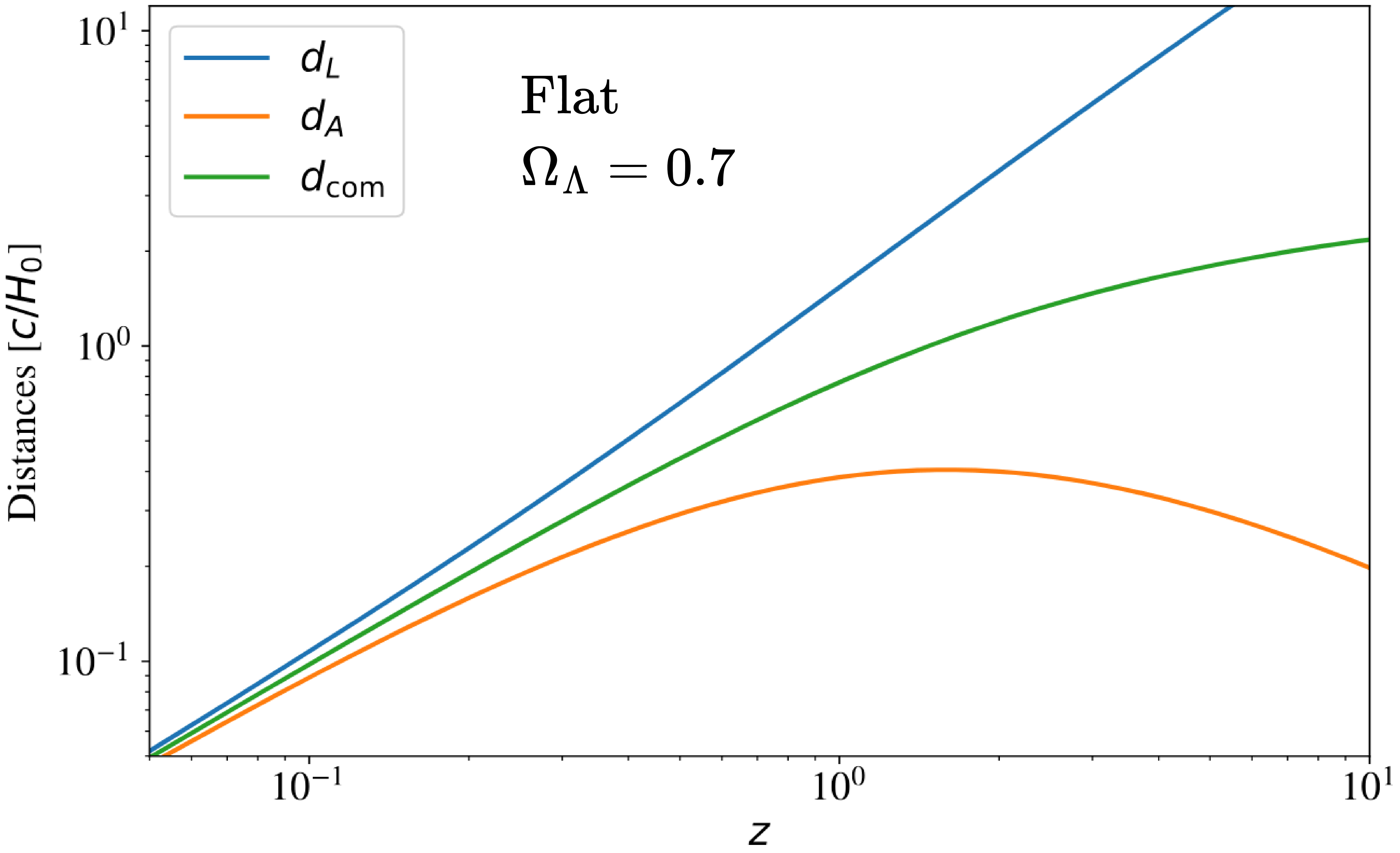
$i =$ each cosmic component

Angular diameter distance

$$d_A = \frac{d_L}{(1+z)^2}$$

Comoving distance

$$d_{\rm com} = \frac{c}{H_0}\int_0^z \frac{{\rm d}z'}{E(z')}$$



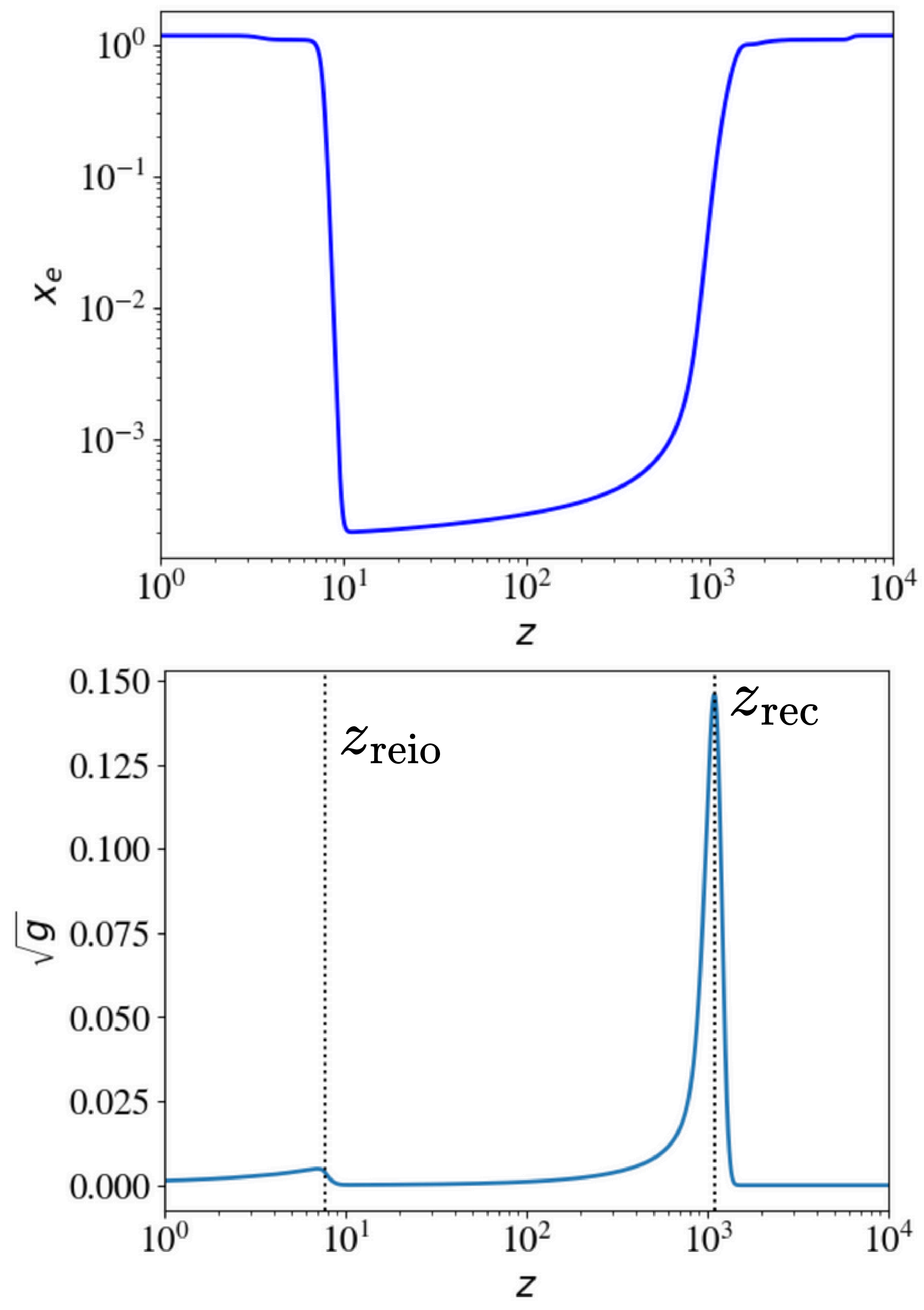
Thermal history

Free electron fraction

$$\frac{dx_e}{dz} = \text{excitation, ionization, heating, ...}$$

Optical depth $\kappa' = \sigma_T a n_p x_e$

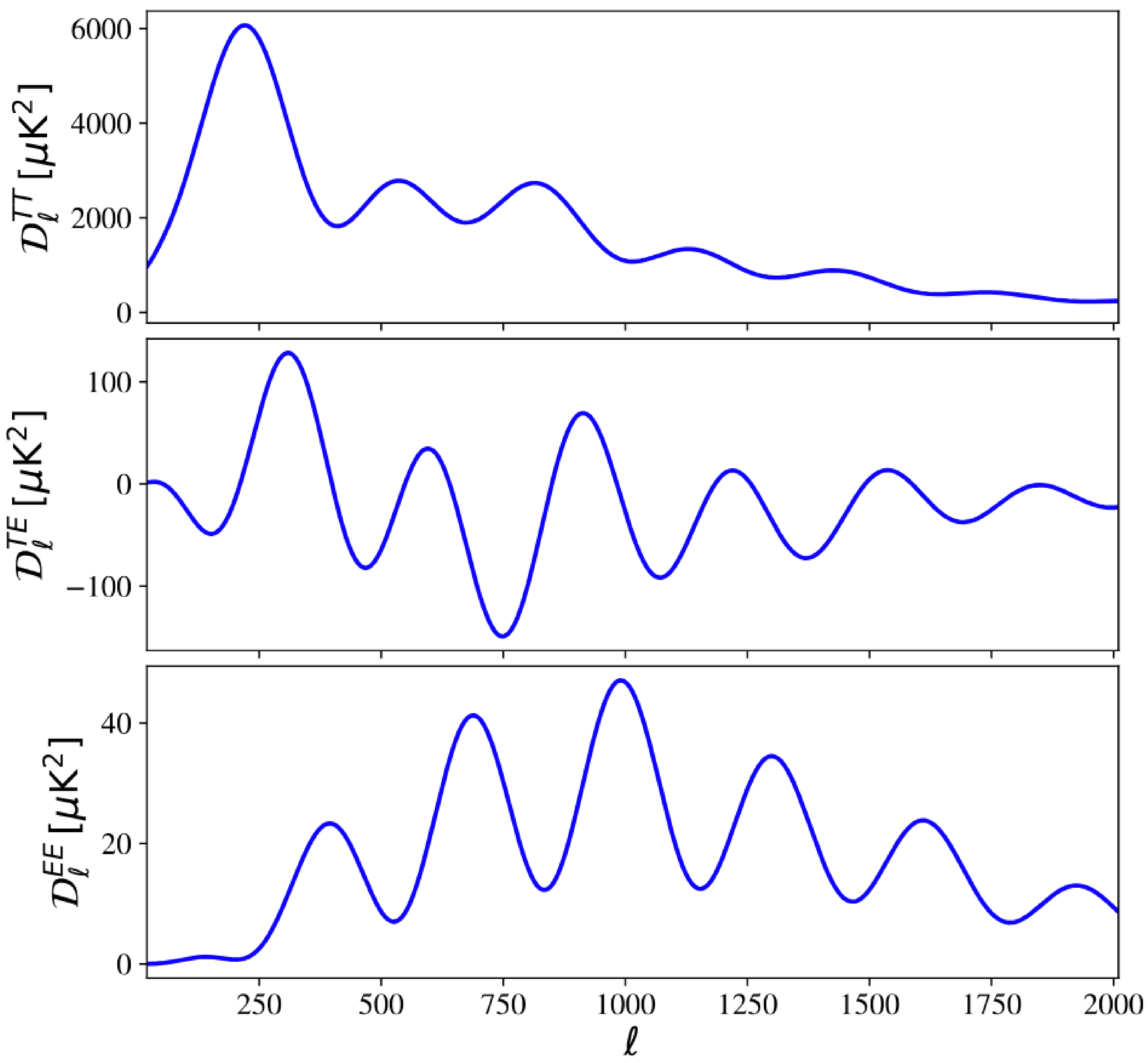
Visibility function $g(\tau) = \kappa' e^{-\kappa}$



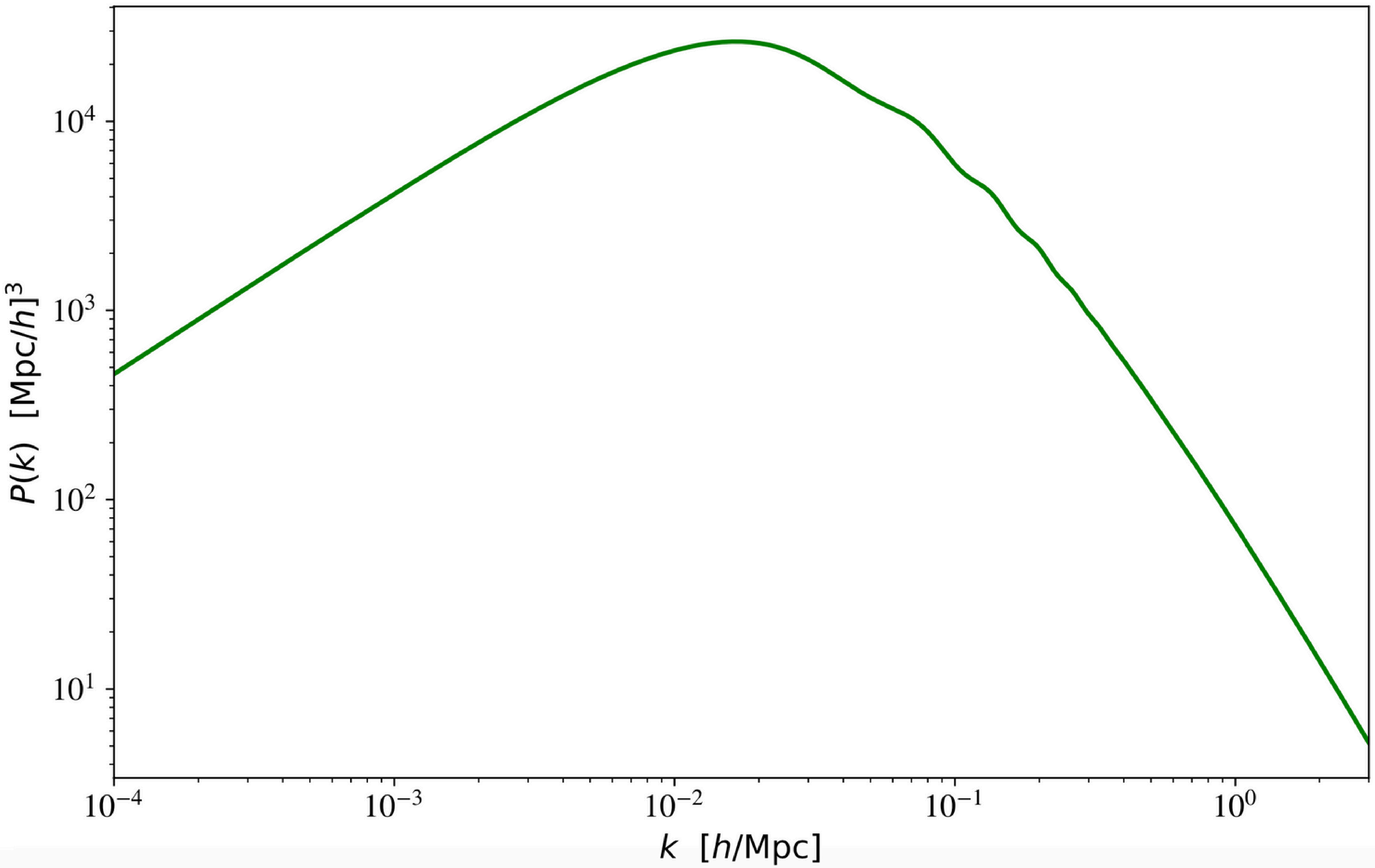
Then $x_e(z) \to \kappa'(z)$ (Thomson scattering rate)
$\to \kappa(z)$ (Optical depth)
$\to \exp(-\kappa(z))$ (factor for Integrated Sachs-Wolfe effect)
$\to g(z)$ (visibility function for Sachs-Wolfe effect)
$\to g'(z)$ (factor for Doppler effect)

CMB spectra

$$C_\ell = 4\pi \int \mathrm{d}\ln k \; j_\ell^2(kD_*)\Delta_T^2(k)$$



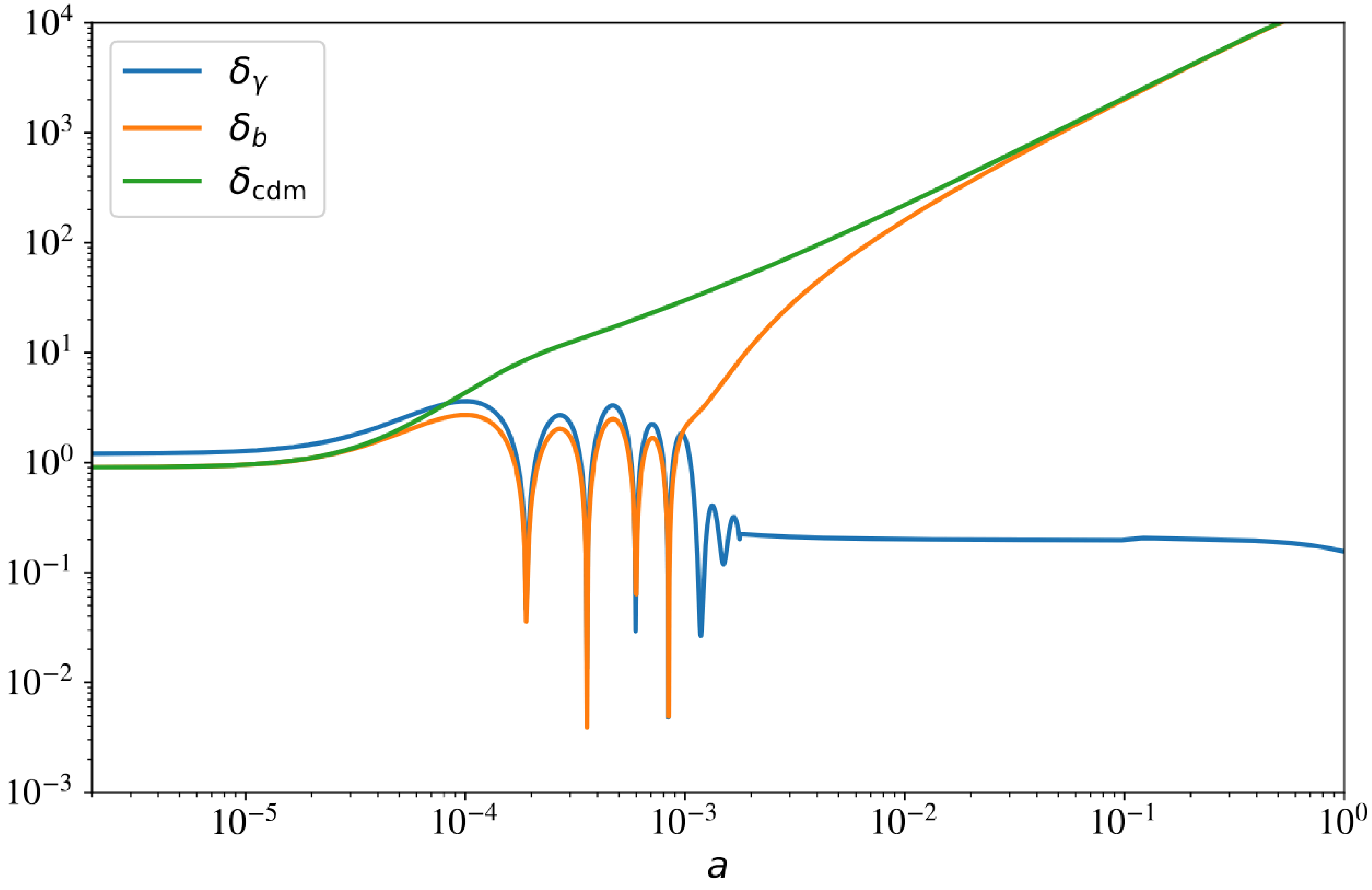
Matter power spectrum



ΛCDM

$z = 0$

Evolution of density and velocity perturbations



Modules in CLASS

module	structure	abv.	*	main content
`input.c`	precision	`pr`	`ppr`	all precision parameters
`background.c`	background	`ba`	`pba`	background quantities as funct. of τ
`thermodynamics.c`	thermodynamics	`th`	`pth`	thermo. quantities as funct. of z
`perturbations.c`	perturbations	`pt`	`ppt`	source (or transfer) functions $S^i(k,t)$
`primordial.c`	primordial	`pm`	`ppm`	primordial spectra $P_R(k), \ldots$
`fourier.c`	fourier	`fo`	`pfo`	2-point statistics (Fourier) $P(k,z), \ldots$
`transfer.c`	transfer	`tr`	`ptr`	harmonic transfer functions $\Delta_l^i(k)$
`harmonic.c`	harmonic	`hr`	`phr`	2-point statistics (harmonic) C_l's
`lensing.c`	lensing	`le`	`ple`	lensed CMB C_l's
`distorsions.c`	distorsions	`sd`	`psd`	CMB spectral distorsions
`output.c`	output	`op`	`pop`	description of output format

Background

Units

CLASS uses the unit $\hbar = c = k_B = 1$, This makes all dimensional quantities having unit in the form Mpc^n for all modules, except thermodynamics.

$$\Rightarrow \text{Conformal time } \tau \text{ in Mpc,} \quad H = \frac{a'}{a^2} \text{ in Mpc}^{-1}$$

$$\Rightarrow \text{All energy densities} \quad \rho_i^{\texttt{class}} \equiv \frac{8\pi G}{3}\rho_i^{\text{physical}} \quad \text{in Mpc}^{-2}$$

$$\Rightarrow \text{Friedmann equation} \quad H = \sqrt{\sum_i \rho_i - \frac{K}{a^2}}$$

New since v3.0: all quantities that should normally scale with some power of a_0^n are renormalised by a_0^{-n}, in order to be independent of a_0, e.g.

- `a` in the code stands for a/a_0 in reality
- `tau` in the code stands for $a_0 \tau c$ in Mpc
- any prime in the code stands for $(1/a_0) d/d\tau$
- `r` stands for any comoving radius times a_0
- etc.

a_0 absorbed everywhere

For example

If you call the Hubble function from CLASS,
it wiill give you the value in the unit of Mpc^{-1} since $c \equiv 1$

So if you want the value of Hubble function at a redshift in the unit of

$$\mathrm{km\,s^{-1}Mpc^{-1}}$$

you have to **multiply it by the speed of light in the unit km/s**

If you prefer the Hubble distance at a redshift in the unit Mpc, just using

$$d_H = \frac{1}{H(z)} \quad \text{in Mpc}$$

background_function()

Energy density: $$\rho_i = \Omega_{i,0} H_0^2 a^{-3(1+w_i)}$$

pressure: $$p_i = w_i \rho_i$$

Hubble function: $$H = \sqrt{\sum_i \rho_i - \frac{K}{a^2}}$$

Critical density: $$\rho_{\text{crit}} = H^2$$

Density parameter: $$\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}}$$

Most quantities can be instantly calculated from a given value of *a*

Perturbation module

Find all perturbations (density, gravitational potential, ...) by integrating ODEs for each independent wavenumber k, each mode (scalar/vector (just in case)/tensor), each initial condition (adiabatic/isocurvature):

- Boltzmann (non-perfect fluids: photon temperature/polarization, massless/massive neutrino temperature)
- Continuity + Euler (perfect fluid: baryons, hypothetical (DE/DM/DR) fluid) or approximatively pressureless species: (CDM)
- linearized Einstein equations (one = differential equation, others = constraint equations)

Linear perturbations ⇒ perturbations normalized to trivial initial condition (class → curvature perturbation $R = 1$ for scalar with adiabatic I.C.)

Perturbation module

Perturbed Einstein equations $\delta G^{\mu}_{\nu} = 8\pi G \delta T^{\mu}_{\nu}$

Metric perturbation (synchroneous gauge) $ds^2 = a^2(\tau)\{-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\}$

solving 2 of the 4 first order Einstein equations:

$$k^2\eta - \frac{1}{2}\frac{a'}{a}h' = -4\pi G a^2 \delta\rho,$$

$$k^2\eta' = 4\pi G a^2(\rho + p)\theta,$$

$$h'' + 2\frac{a'}{a}h' - 2k^2\eta = -24\pi G a^2 \delta p,$$

$$h'' + 6\eta'' + 2\frac{a'}{a}(h' + 6\eta') - 2k^2\eta = -24\pi G a^2(\rho + p)\sigma$$

Together with Boltzmann equation for each cosmic species

Perturbation module

The Boltzmann equation

- At an abstract level we can write:

$$\mathcal{L}\left[f_\alpha(\tau, \mathbf{x}, \mathbf{p})\right] = \mathcal{C}\left[f_i, f_j\right] (= 0). \quad (1)$$

  The last equal sign is true for a **collisionless** species.

- We expand f_α to first order:

$$f_\alpha(\tau, \mathbf{x}, \mathbf{p}) \simeq f_0(q)(1 + \Psi(\tau, \mathbf{x}, q, \hat{n})). \quad (2)$$

- Plugging equation (2) into equation (1) gives a Boltzmann equation for Ψ in Fourier space:

$$\frac{\partial \Psi}{\partial \tau} + i\frac{qk}{\epsilon}(\mathbf{k}\cdot\hat{n})\Psi + \frac{d\ln f_0}{d\ln q}\left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2}(\hat{k}\cdot\hat{n})^2\right] = \mathcal{C}$$

f is the distribution function e.g. Bose-Einstein distribution for CMB photon

T. Tram

Perturbation module

The line-of-sight formalism

The Boltzmann equation has a formal solution in terms of an integral along the line-of-sight:

$$\Theta_l(\tau_0, k) = \int_{\tau_{\rm ini}}^{\tau_0} d\tau \; S_T(\tau, k) \; j_l(k(\tau_0 - \tau))$$

$$S_T(\tau, k) \equiv \underbrace{g\,(\Theta_0 + \psi)}_{\rm SW} + \underbrace{\left(g\,k^{-2}\theta_{\rm b}\right)'}_{\rm Doppler} + \underbrace{e^{-\kappa}(\phi' + \psi')}_{\rm ISW} + {\rm pol.}$$

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Relevant sources
sources for CMB temperature
sources for CMB polarisation
metric perturbations φ and ψ and derivatives, used for lensing and galaxy number counts. density perturbations of all components {δi} velocity perturbations of all components {θi}

Perturbation module

- Solves solves the evolution of all perturbations ($\rightarrow$ Einstein-Boltzmann eq.)
- Stores the source functions $S(k, \tau)$ in structure `perturbs`:
  - sources for CMB temperature
  - sources for CMB polarisation
  - metric perturbations and derivatives (used e.g. for lensing)
  - density perturbations of all components $\{\delta_i\}$
  - velocity perturbations of all components $\{\Theta_i\}$
- When perturbations are integrated, interpolated quantities from thermodynamics and background are used

Transfer module

Purpose of the transfer module

The goal is to compute **harmonic transfer functions** by performing several integrals of the type

$$\Delta_l^X(q) = \int d\tau \; S_X(k(q),\tau) \; \phi_l^X(q,(\tau_0-\tau))$$

for each mode, initial conditions, and several types of source functions. In flat space $k=q$.

Sparse ℓ-sampling

Calculation done for few values of ℓ (controlled by precision parameters). C_ℓ's are interpolated later.

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Spectra module

Computes observable power spectra out of source functions, transfer functions:

Linear matter power spectra

$$P(k, z) = (\delta_m(k, \tau(z)))^2 \mathcal{P}(k)$$

Angular power spectra

$$C_l^{XY} = 4\pi \sum_{ij} \int \frac{dk}{k} \Delta_l^X(k) \Delta_l^Y(k) P(k)$$

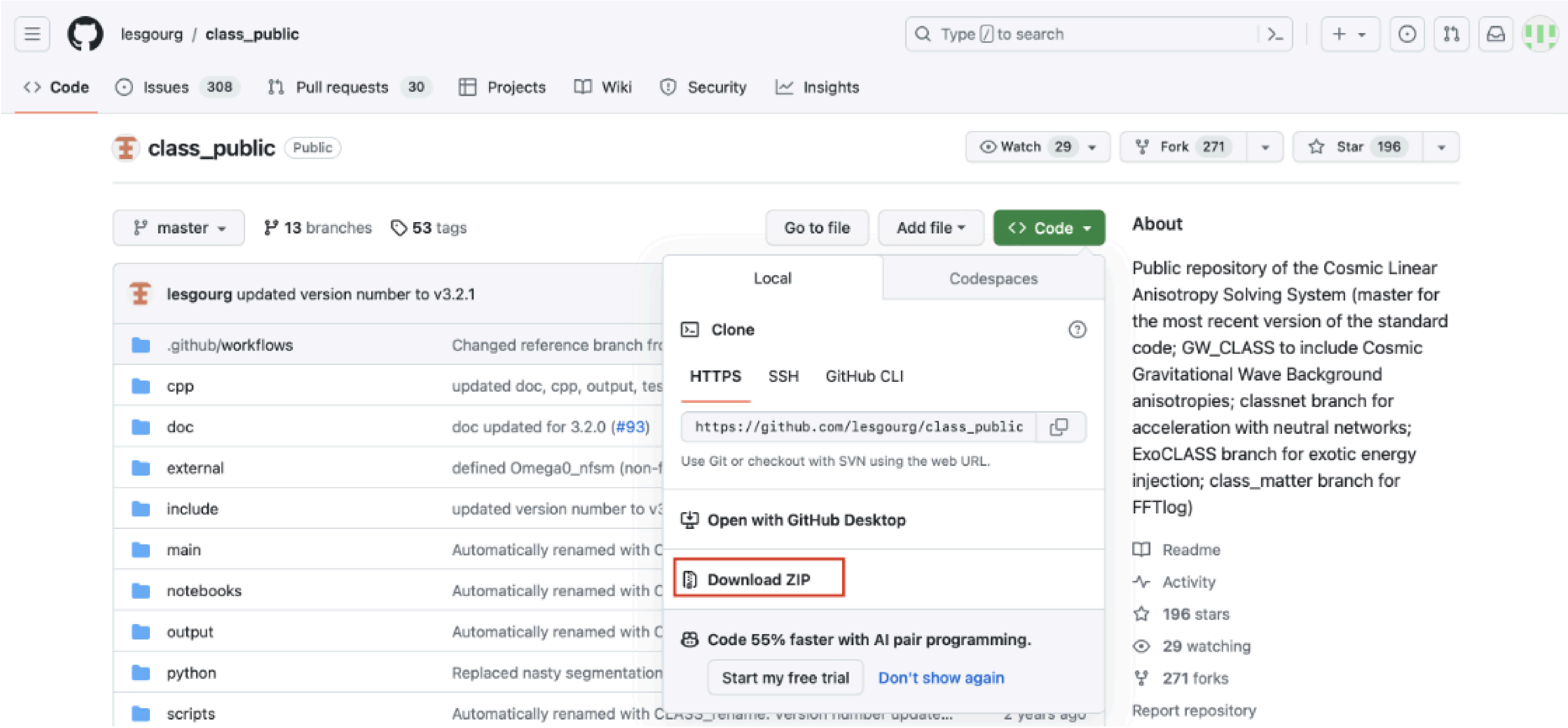
with $XY \in \{TT, TE, EE, BB, PP, TP, \dots\}$

CLASS installation

CLASS can be found on github, you can easily get the CLASS by opening the terminal and then typing

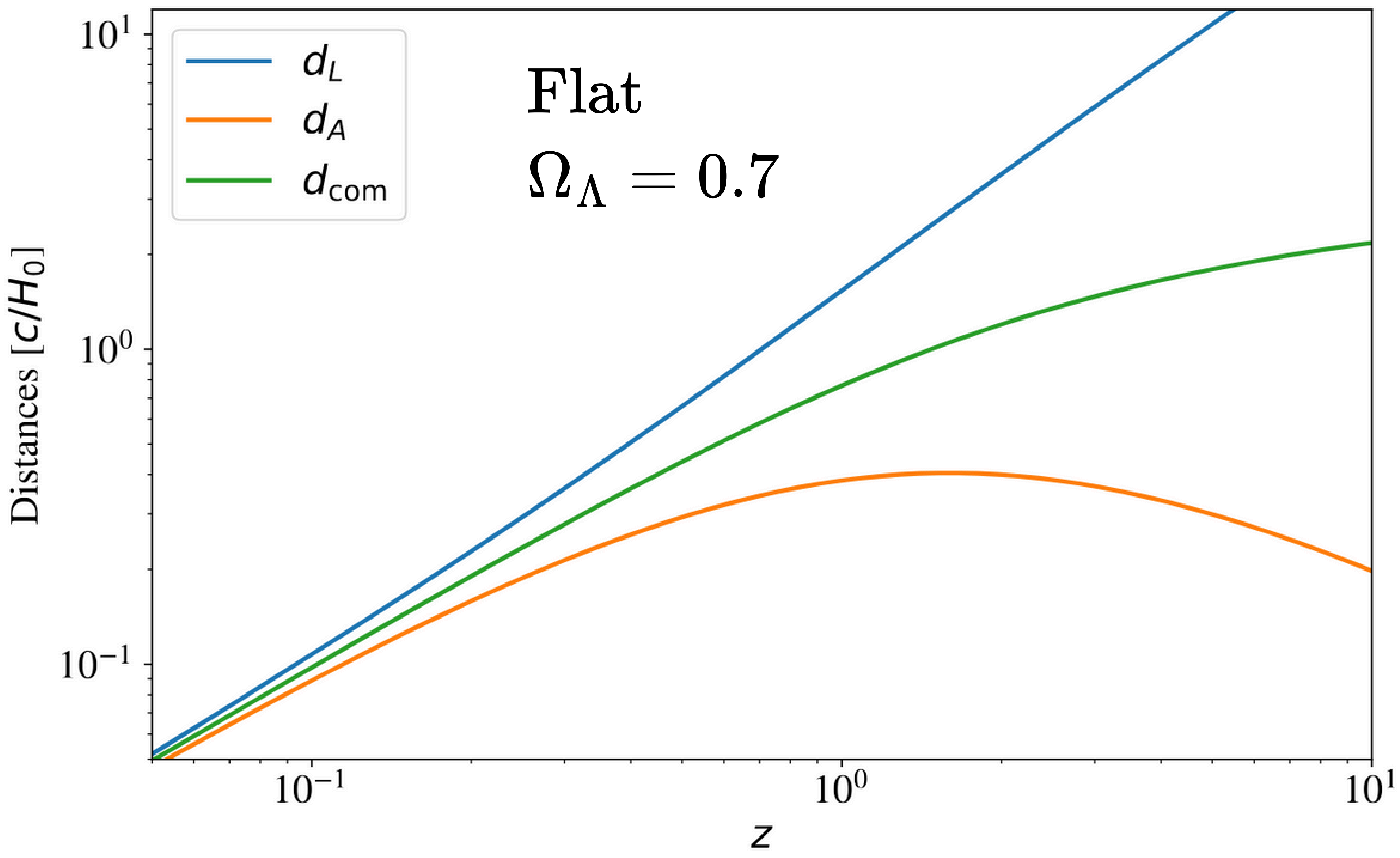
```
$ git clone https://github.com/lesgourg/class_public.git
```

or download directly from https://github.com/lesgourg/class_public



Tutorial exercises

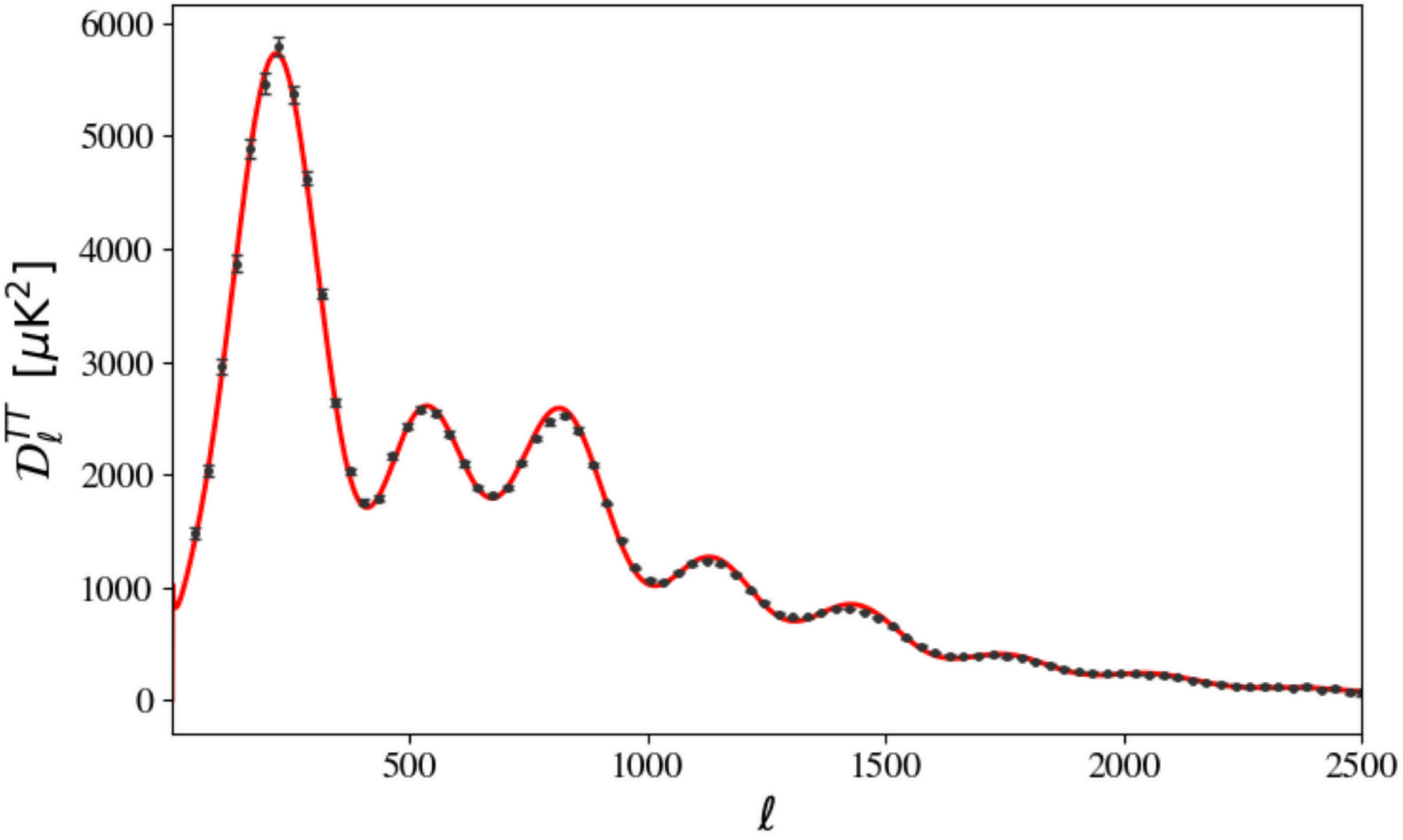
1. Plot the cosmological distances (luminosity, angular diameter distance, comoving distance) using the guiding code below. Compare the result to the Plots that using distance functions calling from CLASS.



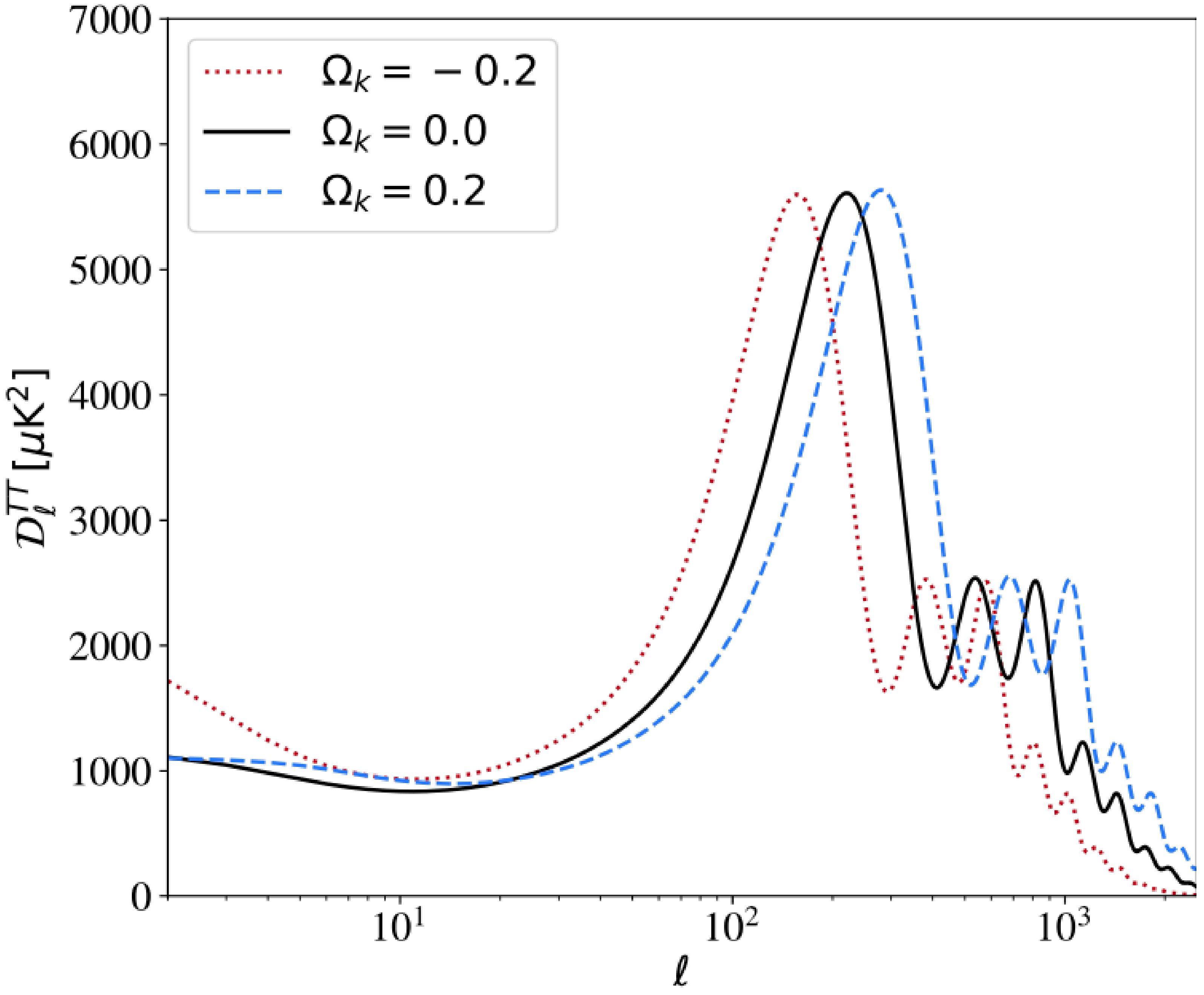
2. Plot the three distances using the Python wrapper for CLASS, comparing your code in the exercise 1

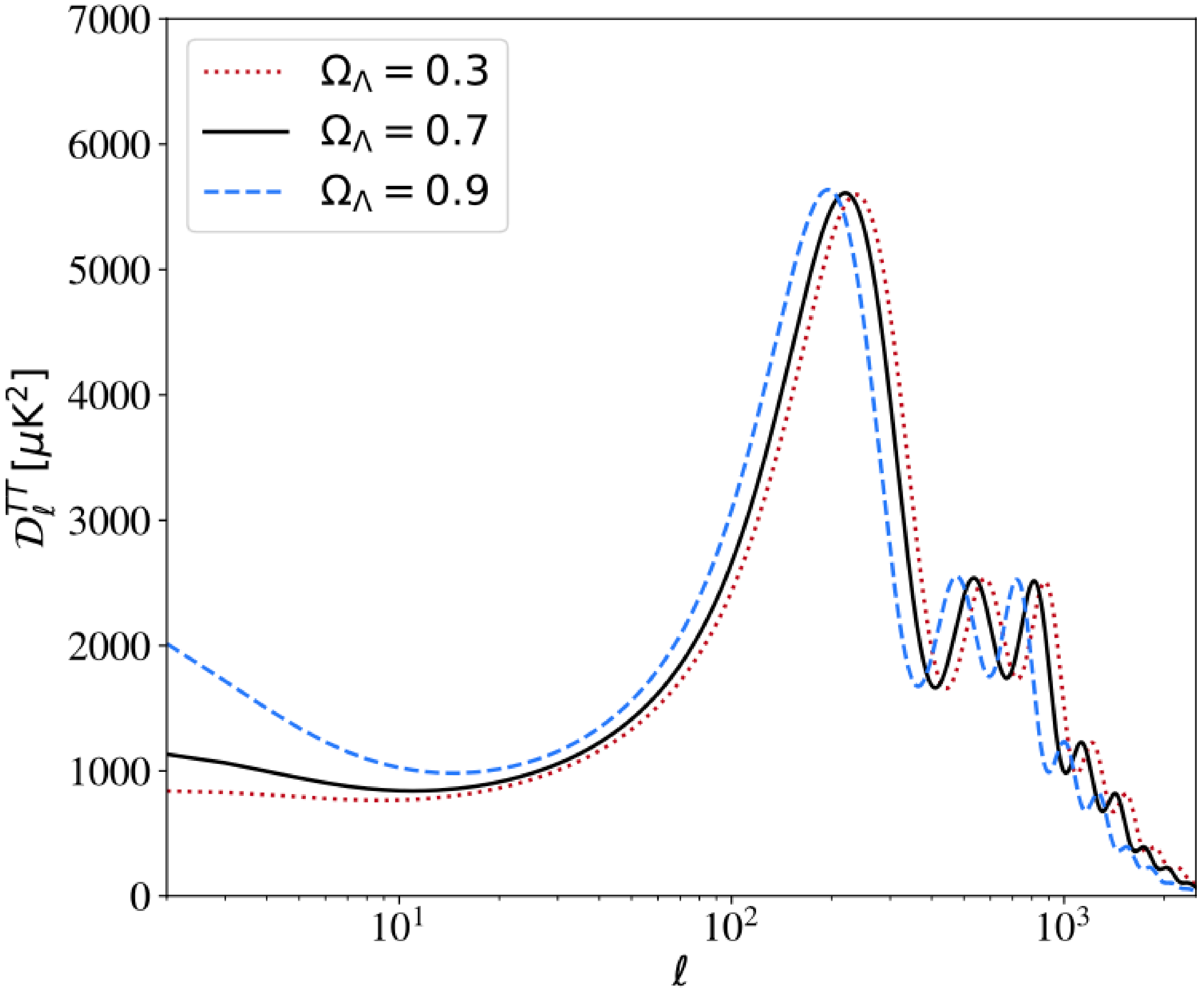
3. Define distance modulus function below (call the luminosity-distance function from CLASS). Plot with three models, $\Omega_m=0.1$, $\Omega_m=0.3$ and $\Omega_m=0.9$ together with the SN Ia observations from the Pantheon+SHOES

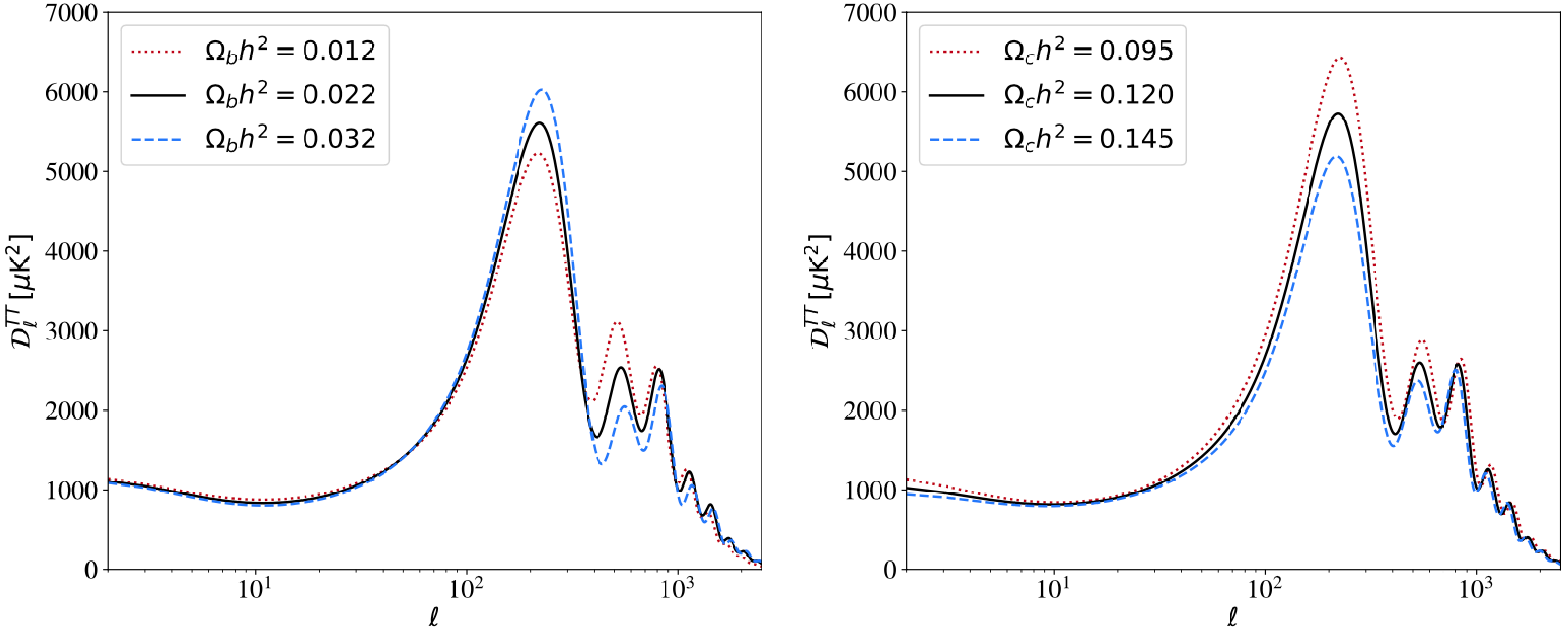
4. Plot the CMB temperature power spectrum againts observational data as shown.



5. Plot CMB spectrum with varying parameters






7000
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell^{TT}\,[\mu K^2]$
$\Omega_b h^2 = 0.012$
$\Omega_b h^2 = 0.022$
$\Omega_b h^2 = 0.032$
10^1
10^2
10^3
ℓ
$\Omega_c h^2 = 0.095$
$\Omega_c h^2 = 0.120$
$\Omega_c h^2 = 0.145$